TOPIC: TOward Perfect InfluenCe Graph Summarization

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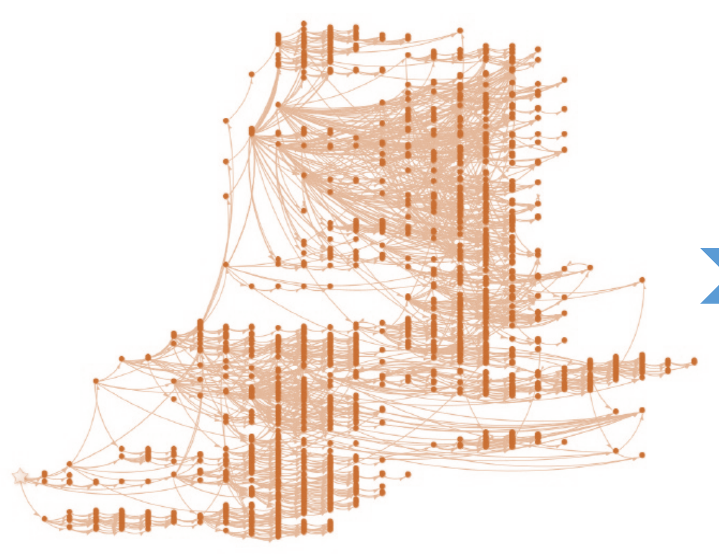
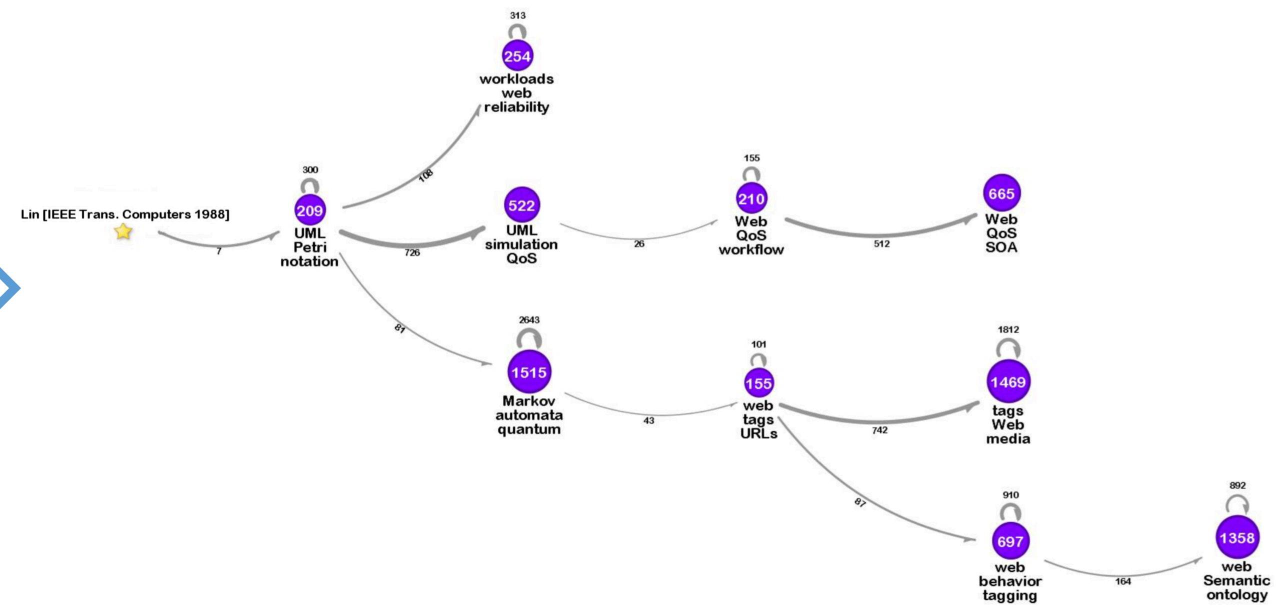
Influence Graph Summarization



Raw Citation Graph



Citation Graph after Summarization

IGS Objective: $\max \sum_{s=1}^{k^2} r(\xi_s)$ which is equal to $\sum_{s=1}^{k^2} \frac{\sum_{v_i \in \pi_{S(\xi_s)}, v_j \in \pi_{T(\xi_s)}} a_{ij}}{\sqrt{|\pi_{S(\xi_s)}||\pi_{T(\xi_s)}|}}$

Motivating Problems on IGS

- Optimality
- Scalability
- Flexibility

New Perspective on IGS Optimality

$$\max \sum_{s=1}^{k^2} r(\xi_s) = \sum_{s=1}^{k^2} \frac{\sum_{v_i \in \pi_{S(\xi_s)}, v_j \in \pi_{T(\xi_s)}} a_{ij}}{\sqrt{|\pi_{S(\xi_s)}||\pi_{T(\xi_s)}|}} = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{a_{ij}}{\sqrt{|\pi_{C(v_i)}||\pi_{C(v_j)}|}}$$

⇩ (as a quadratic form)

$$\max \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j = x^T A x = x^T A^* x \quad \text{where } x = (|\pi_{C(v_1)}|^{-\frac{1}{2}}, \cdots, |\pi_{C(v_n)}|^{-\frac{1}{2}})^T$$

⇩ (by min-max theorem)

$$\max \frac{x^T A^* x}{x^T x} = \lambda_1 \implies x^T A^* x = k \cdot \lambda_1 \implies x = \sqrt{k} q_1$$

⇩ (optimizing the largest component)

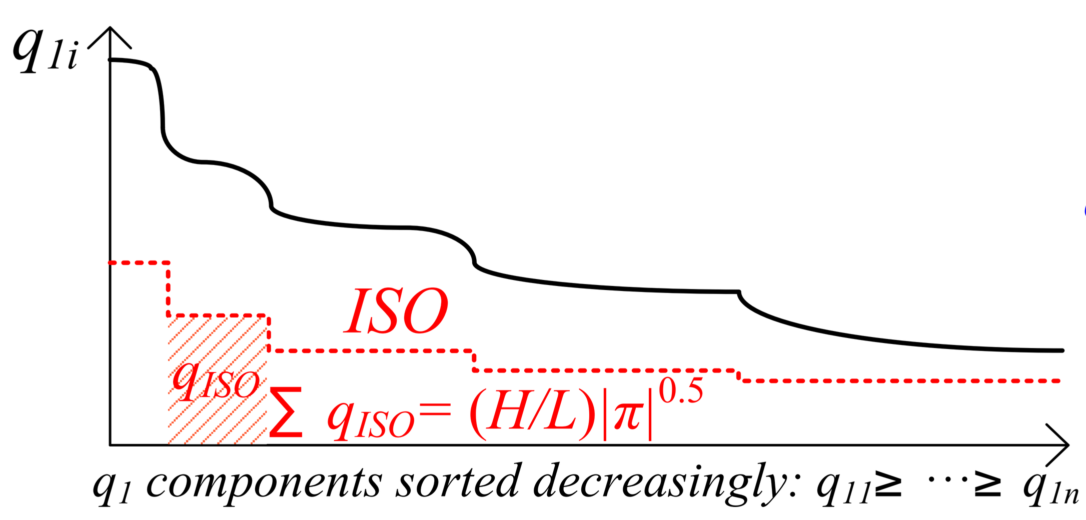
$$\max z_1 = q_1^T x = \sum_{j=1}^{k} |\pi_j|^{-\frac{1}{2}} \sum_{C(v_i)=j} q_{1i}$$

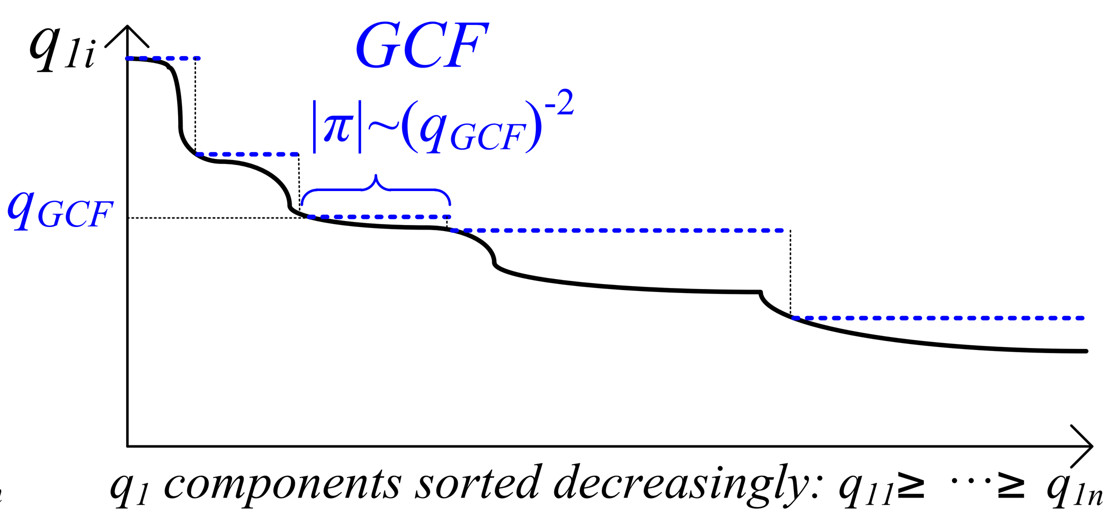
components sorted decreasingly: $q_{11} \geqslant \cdots \geqslant q_{1n}$

$$q_1 = \left[\underbrace{q_{11}}_{\pi_1} \; \underbrace{q_{12}}_{\pi_2} \; \underbrace{\cdots\cdots}_{\cdots\cdots} \; \underbrace{\cdots\cdots \; q_{1n}}_{\pi_k} \right]$$

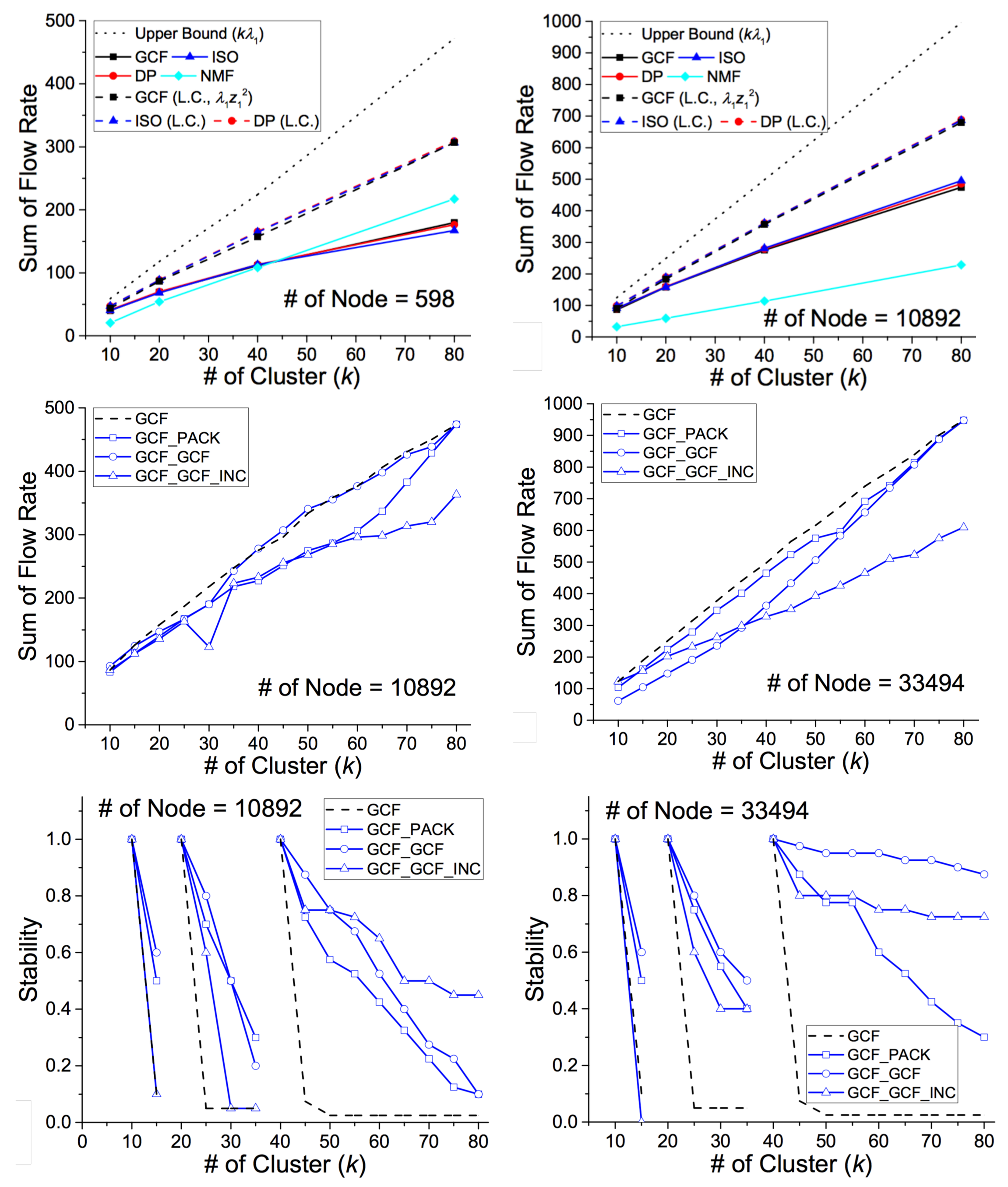
cluster size increasing: $|\pi_1| \leqslant \cdots \leqslant |\pi_k|$

Static Algorithms

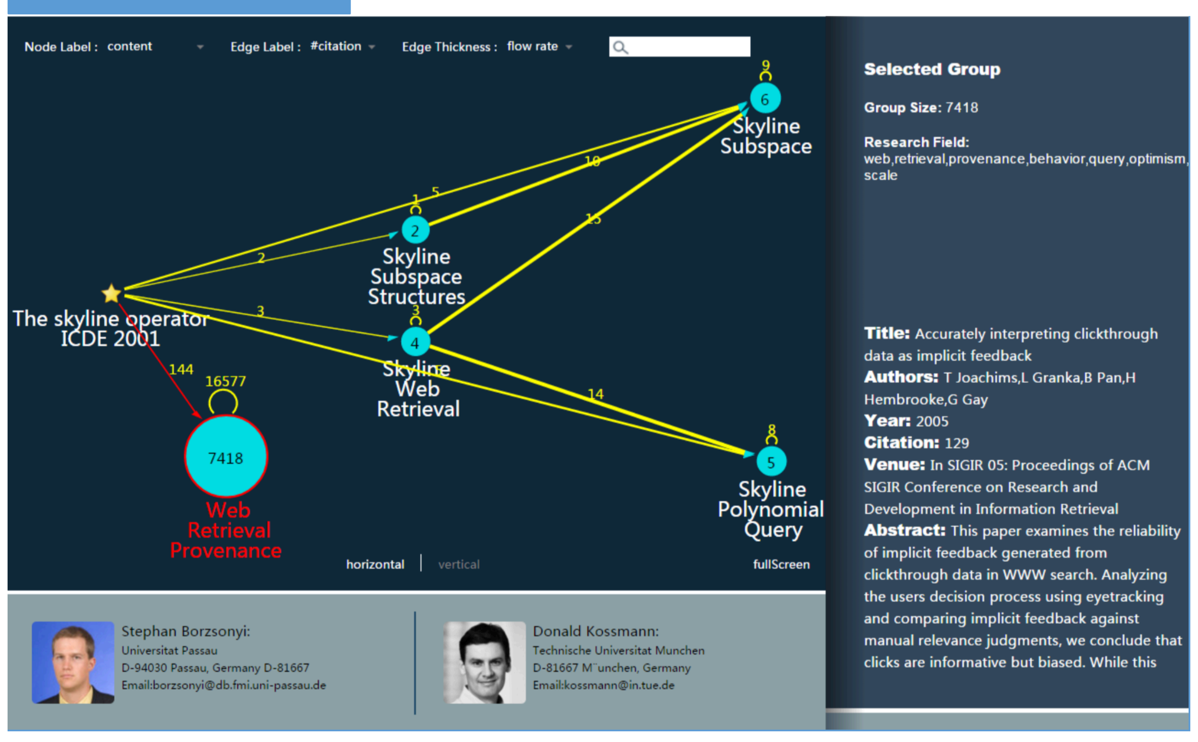




Evaluation



System



Dynamic Algorithm